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# Boundary Operator in the Matrix Product States

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The direct product state is written as  $\prod_i |\psi\rangle_i$ , where  $|\psi\rangle_i$  is a local state at the  $i$ th cluster. To represent quantum entanglement between decoupled clusters, one of natural generalizations is the matrix product state (MPS)  $|\Psi\rangle = \text{Tr} \prod_i A_i$  with matrix elements  $(A_i)_{mm'} = |\psi_{i;mm'}\rangle_i$ . The translationally invariant MPS under the periodic boundary condition in one dimensional systems is written as  $|\Psi\rangle = \text{Tr} \prod_i A$  with a single uniform matrix  $A$ . To include the boundary effect, one can consider the boundary matrix  $Q$  with matrix elements  $(Q)_{mm'} = |\phi_{mm'}\rangle_0$  and  $|\Psi\rangle = \text{Tr} [Q \prod_i A]$ , where the artificial Hilbert space  $|\phi\rangle_0$  is set to be one-dimensional generally [1]. Does not the translationally invariant MPS have the boundary operator  $Q$ ?

Our studies show the importance of  $Q$  for the MPS. We have derived a MPS representation of the Bethe ansatz state for spin-1/2 Heisenberg chain [2] and the Lieb-Liniger model [3], from the algebraic Bethe ansatz using the factorizing  $F$ -matrices. The uniform matrix  $A$  obtained for the Heisenberg chains is the same as that in the matrix product ansatz [4] apart from normalization factors. For the Lieb-Liniger model describing the Bose gas with delta-function interaction in one-dimension, a “continuous” extension of the matrix product state is obtained. The exact MPS has both translationally invariance and the boundary operator  $Q$ . The latter comes from the domain wall boundary conditions [5]. In fact, for the MPS in the Bose gas,  $Q$  plays a role in fixing the number of particles. From a numerical point of view,  $Q$  is also important to consider the spontaneous symmetry breaking of the translational symmetry and long-period super-lattices for the magnetic plateau [6].

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